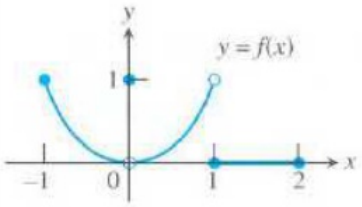
What you'll Learn About

- Continuity at a point
- Continuous Functions
- Intermediate Value Theorem for Continuous Functions



1*a*. Does $f(-1)$ exist? yes $f(-1)=1$

1*b*. Does $\lim_{x\to -1^+} f(x)$ exist? yes $\lim_{x\to -1^+} f(x)=1$

1*c*. Does $\lim_{x\to -1^+} f(x) = f(-1)$ yes

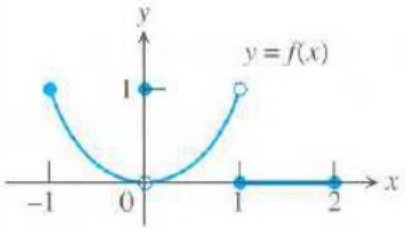
1*d*. Is f continuous at x = -1? yes

Definition of continuity at an endpt.

Not Continuous

Hole
Jump
VA

Definition of continuity ←



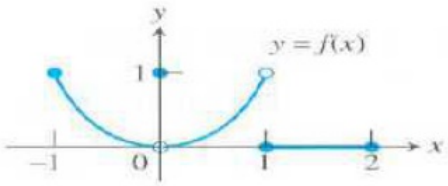
2a. Does $f(0)$ exist? yes $f(0)=1$

2b$_i$. Does $\lim_{x\to 0^+} f(x)$ exist? $\lim_{x\to 0^+} f(x)=0$

2b$_{ii}$. Does $\lim_{x\to 0^-} f(x)$ exist? $\lim_{x\to 0^-} f(x)=0$

2c. Does $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$ NO

2d. Is f continuous at x = 0? NO



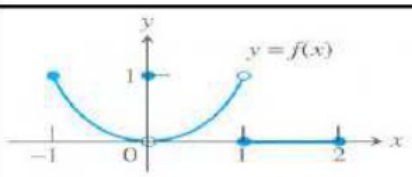
3a. Does $f(1)$ exist? $f(1)=0$

3b$_i$. Does $\lim_{x\to 1^+} f(x)$ exist? $\lim_{x\to 1^+} f(x)=0$

3b$_{ii}$. Does $\lim_{x\to 1^-} f(x)$ exist? = 1

3c. Does $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(1)$ NO

3d. Is f continuous at x = 1? NO

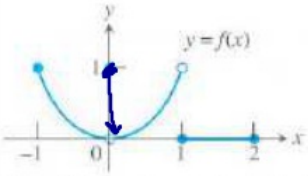


4a. Does $f(2)$ exist?

4b. Does $\lim_{x \to 2} f(x)$ exis

4c. Does $\lim_{x \to 2^-} f(x) = f(2)$

4d. Is f continuousat $x = 2$?



Hole
Removeable
discontinuities

Non-Removeable
Jump
VA

5. For what values is the function continuous

$[-1, 0) \cup (0, 1) \cup (1, 2]$

6a. Is it possible to extend f to be continuous at x = 0? If so, what value should the extended function have? If not, why not?

yes $f(0) = 0$

6b. Is it possible to extend f to be continuous at x = 1? If so, what value should the extended function have? If not, why not?

NO

Determine the type of discontinuity

A) $f(x)=\begin{cases} 3+x & x<2 \\ 1 & x=2 \\ \frac{x}{2} & x>2 \end{cases}$

$f(2)=1$

$\lim_{x\to 2^-} f(x)=5$ $\lim_{x\to 2^+} f(x)=1$

Jump (Non Removable)

B) $f(x)=\begin{cases} \frac{1}{x-2} & x<2 \\ x^2+5x & x>2 \end{cases}$

$f(2)=DNE$

$\lim_{x\to 2^-} f(x)=-\infty$ $\lim_{x\to 2^+} f(x)=14$

V.A (Non removable)

C) $f(x)=\begin{cases} 9-x^2 & x\neq 3 \\ 5 & x=3 \end{cases}$

$f(3)=5$

$\lim_{x\to 3^-} f(x)=0$ $\lim_{x\to 3^+} f(x)=0$

Hole (Removable)

D) $f(x)=\begin{cases} 6-x & x<3 \\ 2x-3 & x>3 \end{cases}$

$f(3)=DNE$

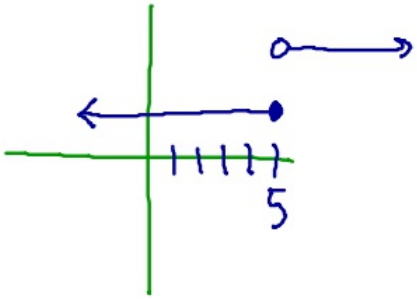
$\lim_{x\to 3^-} f(x)=3$ $\lim_{x\to 3^+} f(x)=3$

Hole (Removable)

Given the following information, sketch a graph of f(x)

A) f(x) exists, but $\lim_{x\to 5}$ does not

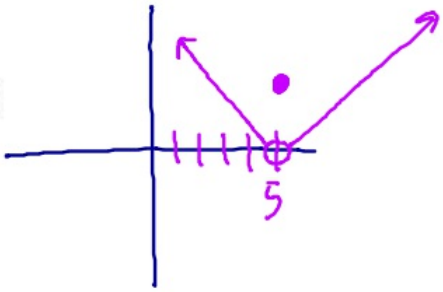
defined everywhere



B) f(5) exists

$\lim_{x\to 5}$ exists

f is not continuous at $x = 5$



Find a value for a so that the function is continuous

47) $f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \geq 3 \end{cases}$

$x^2 - 1 = 2ax$

$3^2 - 1 = 2a(3)$

$\frac{8}{6} = \frac{6a}{6}$

$a = \frac{8}{6} = \frac{4}{3}$